In a nutshell: The finite-difference method for linear ordinary differential equations with constant coefficients

Given a second order linear ordinary differential equation with constant coefficients

$$a_2 u^{(2)}(x) + a_1 u^{(1)}(x) + a_0 u(x) = g(x),$$

two spatial boundary points $[a, b]$ and two boundary values $u(a) = u_a$ and $u(b) = u_b$.

Parameters:

n The number of sub-intervals into which $[a, b]$ will be divided.

1. Set $h \leftarrow \frac{b-a}{n}$ and $x_k \leftarrow a + kh$ noting that $x_n = b$.
2. Set

$$\begin{aligned} p &\leftarrow 2a_2 - a_1 h \\ q &\leftarrow -4a_2 + 2a_0 h^2 \\ r &\leftarrow 2a_2 + a_1 h \end{aligned}$$

3. Create and solve the system of $n - 1$ linear equations in $n - 1$ unknowns

$$\begin{pmatrix} q & r & & & & & \\ p & q & r & & & & \\ & p & q & r & & & \\ & & p & q & r & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & p & q & r \\ & & & & & p & q \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \vdots \\ u_{n-2} \\ u_{n-1} \end{pmatrix} = \begin{pmatrix} 2g(x_1)h^2 - pu_a \\ 2g(x_2)h^2 \\ 2g(x_3)h^2 \\ 2g(x_4)h^2 \\ \vdots \\ 2g(x_{n-2})h^2 \\ 2g(x_{n-1})h^2 - ru_b \end{pmatrix}$$

4. The approximation of $u(x_k)$ is u_k for $k = 1, \ldots, n - 1$ and $u(x_0) = u(a) = u_a$ and $u(x_n) = u(b) = u_b$